Evaluating optimal purchasing and selling decisions of beef cattle replacement females

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Abstract

Purpose – The objective of this research was to determine the optimal age and pregnancy status for buying and selling replacement of beef females for risk-neutral and risk-averse producers.

Design/methodology/approach – A hedonic pricing model was estimated to measure how age, pregnancy status, breed and cull cow prices impact the sale price of these cattle. Data came from an annual heifer and cow sale in Tennessee between 2009 and 2018. A financial simulation model was developed to generate distributions of net present value (NPV) for buying replacement females at various ages and pregnancy status and then selling that female at various ages and pregnancy status.

Findings – The hedonic pricing model indicates sale prices were highest for five-year-old cows that were between four to five months pregnant. NPV was higher for buying heifers versus buying cows and for buying an open female versus a pregnant female. Regardless of age and pregnancy status when purchased, NPV was higher when the female was sold as pregnant prior to the end of her productive life. The risk analysis showed that risk aversion, buying older open cows and selling them as pregnant earlier in their productive life was preferred.

Originality/value – This research offers unique insight into how pregnancy status and age at sale impacts the animal’s NPV while considering risk. These results have implications for educating producers on purchasing and selling decisions of heifers and cows as well as for lenders who finance these purchases.

Keywords Beef cattle, Hedonic pricing, Net present value, Replacement decision, Simulation

Paper type Research paper

Introduction

The decision of when to sell and replace a beef cow has been the focus of many economic studies over the last several decades (Bentley et al., 1976; Melton, 1980; Matthews and Short, 2001; Trapp, 1986; MacKay et al., 2004; Ibendahl et al., 2004). MacKay et al. (2004) explored several marketing strategies for female cattle and showed that when cattle prices are high, selling bred yearlings and retaining heifers to develop was most profitable. However, in times of low cattle prices, profits were maximized by selling open heifers and retaining bred yearlings. Ibendahl et al. (2004) concluded the cost of development and production can influence the timing of selling an open cow. They reported that during times of high feed costs, keeping open cows was advantageous compared to developing heifers to replace cows. Results generally show the profit-maximizing decision primarily depends on cattle prices and development costs (Mathews and Short, 2001; Ibendahl et al., 2004; Mackay et al., 2004), meaning the optimal decision is dynamic and changes annually.

Hedonic pricing models have been used to understand what drives the price of replacement females purchased in an auction (Parcell et al., 1995; Elliott et al., 2013; Mitchell et al., 2018). Parcell et al. (2006) found calf performance and carcass quality expected progeny differences impacted a heifer’s value. They also found that heifers with expected calving dates in early spring (January and February) brought higher prices than heifers anticipated to calve in later spring (March and April). Furthermore, Mitchell et al. (2018) showed pregnant cow prices increased until they were eight months pregnant. Cows further along in pregnancy (sometimes called long bred) were assumed to be less likely to lose a calf, have a lower
production cost to calving and create revenue more quickly. They also found cow prices increased with cow age until the age of five, when prices then started decreasing.

These studies are insightful and have produced impactful results for producers. We extend this research by determining optimal age and pregnancy status for buying replacement females given their age and pregnancy status at the time of their sale. We first estimate a hedonic pricing model to measure how age, pregnancy status, breed and cull cow prices impact the sale price of these cattle. Then, a net present value (NPV) simulation model was established to generate distributions of NPV for buying and then selling replacement females at various ages and pregnancy status.

This research contributes to this body of the literature by exploring the buying and selling ages and pregnancy status that produces the greatest long-term profitability. The risk analysis of these different scenarios provides unique insight into how a risk-averse producer’s optimal decision might differ from a risk-neutral producer. Results can also be a useful demonstration to producers when making these investment decisions as well as bankers who are lending money for these purchases.

**Economic framework**

A common approach to analyze the purchase of replacement females in a beef cattle herd is NPV, which is the sum of the discount value of future net returns (Bentley et al., 1976; Melton, 1980; Matthews and Short, 2001; Trapp, 1986; Mackay et al., 2004; Ibendahl et al., 2004; Boyer et al., 2020). This is because purchases of replacement females have a one-time investment cost, future streams of returns from calf sales and a salvage value at the end of their productive life. Net returns are the production costs, which might include forage, land, animal health, labor, breeding and harvested feedstuffs, subtracted from the revenue generated from the selling of calves. Revenue is, therefore, determined by cattle prices and calf weights when they are sold. Assuming a producer buys replacement, females and sells her calf before finishing, annual net returns can be expressed mathematically as

\[
E[\pi_t] = p_s^t y_s^t \left( \frac{CR}{2} \right) + p_h^t y_h^t \left( \frac{CR}{2} \right) - PC_t
\]

where \( \pi_t \) is the expected annual net returns ($/head) in time period \( t (t = 1, \ldots, T) \); \( p_s^t \) is the price of steer calves ($/lb); \( y_s^t \) is the weight of the steer calves (lb/head); CR is the calving rate \( 0 \leq CR \leq 1 \); \( p_h^t \) is the price of the heifer calves ($/lb); \( y_h^t \) is the weight of heifer calves (lb/head); and \( PC_t \) is the annualized production costs for each calving herd ($/head).

The annual net returns over the timeframe the female remains in the herd \( (T) \) are discounted to find the NPV which measures the profitability of an investment over multiple years of investment generating revenue. Included in the NPV is the salvage value or sometimes called terminal value of the female when the producer chooses to sell her. This can occur at the end of her productive life or while she might still be productive. NPV is generally expressed mathematically as

\[
E[NPV] = -IC_1 + \sum_{t=1}^{T} \frac{\pi_t}{(1+R)^t} + SV_T \left( \frac{1}{1+R} \right)^T
\]

where \( NPV \) is the sum of the discounted annual net returns; \( R \) is the risk-adjusted discount rate; \( IC_1 \) is the investment cost or the purchase price of the female ($/head); and \( SV_T \) is the salvage value or the price received when the female is sold ($/head). If the female is not pregnant when purchased, this initial investment cost would also include breeding expenses in the first year.
For this analysis, we are interested in exploring how NPV changes when replacement females are bought and sold at various ages and stages of pregnancy. For example, we compare the profitability of purchasing a non-pregnant (or open) heifer and then selling her as a pregnant five-year old cow versus buying an open heifer and selling her as an open cow at the end of her productive life or terminal age. This analysis also considers how NPV is impacted by buying a bred female and selling her as an open female, which can show the importance of reproductive management.

The scenario with the highest NPV would be the preferred scenario for a risk-neutral individual. When considering risk, the decision-maker prefers the scenario with this highest utility, which can be defined as $U(NPV, r)$ where $r$ is the individual’s risk preference level. A risk-averse producer would be willing to take a lower expected NPV with certainty instead of a higher expected NPV with uncertainty. This means a risk-averse producer would select the age and pregnancy status at the purchase and sale of the animal with the highest certainty equivalent (CE) at a given risk aversion level (Chavas, 2004).

**Methods**

Hedonic pricing modeling was estimated to determine the impact of age, pregnancy status, breed and cull cow prices on the selling price of replacement females. Predicted values from the hedonic pricing model for females at various ages and pregnancy status will be used for the investment cost and salvage value. Next, we will use a weaning weight response function to cow age from Boyer et al. (2020) to build a hypothetical NPV simulation model to compare distributions across the scenarios.

**Hedonic pricing estimation**

A hedonic pricing model was used to determine the impact of age, pregnancy status, cull cow price, artificially inseminated (AI)-sired and breed on the sale price of female beef animals. We specify a log-level model by taking the log of sale price, correcting the non-normality issue (Wooldridge, 2013). The model is written as

$$\ln(Purchase_{it}) = \beta_0 + \beta_1 M_i + \beta_2 M_i^2 + \beta_3 A_i + \beta_4 A_i^2 + \beta_5 (M_i \times A_i) + \beta_6 AI_i + \beta_7 CP_t$$

$$+ \beta_8 AG_i + \beta_9 GV_i + \beta_9 BL_i + v_t + u_i + \epsilon_{it}$$

(3)

where $Purchase_{it}$ is the purchased price of the $i$th lot of female animals in time period $t$; $M_i$ is the average months pregnant of the lot at the time of the sale; $A_i$ is the average age of the lot sold (in years); $AI_i$ is an indicator variable equal to one if the pregnancy was AI-sired and zero if a natural service pregnancy; $CP_t$ is the monthly price of cull cows in Tennessee at the time of the sale ($/cwt$); $AG_i$ is an indicator variable equal to one if the lot was either defined as Angus and zero otherwise; $GV_i$ is an indicator variable equal to one if the lot was either defined as Gelbvieh and zero otherwise; $BL_i$ is an indicator variable equal to one if the lot was either defined as Balancer and zero otherwise; $\beta$’s are parameters to be estimated; $v_t \sim N(0, \sigma_v^2)$ is the year random effect; $u_i \sim N(0, \sigma_u^2)$ is the lot number random effect; and $\epsilon_{it} \sim N(0, \sigma_e^2)$ is the random error term. Independence is assumed across all random components.

Recently, non- or semi-parametric estimations of livestock hedonic pricing models have been found to be preferred to traditional parametric approaches (Tang et al., 2020). These modeling approaches are applicable when testing parameter estimates over time or across space. However, we use the traditional parametric approach, which is like previous studies (Parcell et al., 1995; Elliott et al., 2013; Mitchell et al., 2018), since we are not comparing prices.
across time or space. This would be an interesting methodological application for a future study to compare prices of females over time.

We can solve for the length into pregnancy and age that maximizes sale price by taking the first-order conditions of equation (3) with respect to these variables and solving for the respective variable. Parameter estimates are substituted into equation (2) for the estimated investment cost and salvage value. Since we take the log of the dependent variable, parameter estimates can also be converted to dollars by multiplying the parameter estimated by the average predicted selling price of the females in the sample.

Heteroscedasticity is a common problem for estimating cattle hedonic pricing models (Mitchell et al., 2018). The likelihood ratio test was used to determine if heteroscedasticity was present from year, lot size and breed. If heteroscedasticity was present, we corrected it using multiplicative heteroscedasticity in the variance equation (Wooldridge, 2013). The model was estimated using maximum likelihood with the MIXED procedure in SAS 9.4.

Simulation
Future revenue streams generated from purchased females are uncertain. Variability of weaning weights was incorporated following Boyer et al.’s (2020) response function of dam age and calf sex. This quadratic response function demonstrates that weaning weights increased until cows were seven years old and then weaning weights began decreasing. They also included random effects that control for unobserved heterogeneity for year and sire. The response parameters in Boyer et al. (2020) were drawn from the multivariate normal (MVN) distribution and were incorporated in equation (1). Appendix shows the equation, simulation methods and results from Boyer et al. (2020). Prices for steers and heifers were randomly drawn from a multivariate empirical distribution derived using historical Tennessee price data.

Scenarios included in this analysis allowed producers to buy a heifer or females that were two-, three- or four-years old. These females could be either open (not pregnant) or pregnant (five months pregnant). We allowed the producer to sell these females at ages five, seven, nine or 12 years old, and they were assumed to be either open or five months pregnant if they were less than 12 years old. All 12-year old females were assumed to be sold open and at the end of their productive life (T = 12).

Two assumptions were made about the producer’s managerial decisions. One is calves are sold at weaning (or soon after weaning). Second, the data used for the hedonic pricing model comes from an annual November sale (more detail is provided below) and not year-round. Due to this data limitation, we cannot consider the seasonality of female prices in our model. We recognize this to be a potential shortcoming of this study. Therefore, we cannot consider calving season in this study.

Simulation and Econometrics to Analyze Risk (SIMETAR©) was used to develop the distributions and perform the simulations (Richardson et al., 2008). A total of 1,000 net return observations were simulated for each scenario.

Risk analysis
We originally used stochastic dominance to compare the NPV cumulative distribution functions (CDFs) for all scenarios but this approach was unable find dominate scenario. Therefore, we used stochastic efficiency with respect to a function (SERF) to rank the scenarios over a range of absolute risk aversion if stochastic dominance does not find a solution (Hardaker et al., 2004, 2015). This requires specifying a utility function, \( U(\text{NPV}, r) \). We use a negative exponential utility function, which specifies a constant absolute risk-aversion coefficient (ARAC) to calculate the CE (Pratt, 1964). The ARAC is found by dividing the derivatives of the person’s utility function \( r_a(r) = -U''(r)/U'(r) \). There are several
advantages to using the negative exponential utility function, which is why Hardaker et al. (2004, 2015) recommends this functional form, but this function is not without limitations (Hardaker et al., 2015). Following Hardaker et al. (2004), a vector of CEs were derived, bounded by a low and high ARAC. The lower bound ARAC was zero, which assumes the producer was risk neutral and a profit-maximizer. The upper bound ARAC was found by dividing four by the expected NPV for all scenario, which indicates extreme aversion to risk. ARAC values in this study ranged from 0.0 for risk neutral to 0.05 for extremely risk averse. The SERF analysis were also conducted in SIMETAR© (Richardson et al., 2008).

The difference between CEs of any two scenarios gives a utility weighted risk premium, which is the minimum dollar amount a producer would require to switch from the scenario with the greatest CE to the alternative scenario with the lesser CE. Risk analysis results are presented and discussed in terms of risk premiums.

**Data**

**Hedonic pricing**

Data were from an annual cow and heifer sale in Crossville, Tennessee that occurs in November at the University of Tennessee Plateau Research and Education Center. There was ten years of data from 2009 to 2018. The data contains information on whether the animal was open or how far along she was into pregnancy. Table 1 shows the summary statistics of the data used in this study. Over this period, a total of 241 lots of females were sold with the number of lots each year ranging from 20 to 32 with an average of 24 lots sold each year. Females were sold in lots ranging from one to four head with most lots being two and three head.

The average sale price over this time period was $1,616/head and prices ranged from around $700 to $3,700/head. Cull cow prices were assumed using monthly Tennessee price data for 80–85% boning cow prices over this same time period (United States Department of Agriculture (USDA) Agricultural Marketing Service (AMS), 2018). All prices were adjusted into 2018-dollar values using the Implicit Gross Domestic Product Price Deflator (US Department of Labor, 2018).

The average age of the females sold was between four and five years old with the youngest being an open heifer not even a year old and the oldest being 11 years old. Pregnancy status

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>SD</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average real price per lot ($)/head</td>
<td>1616.88</td>
<td>609.13</td>
<td>697.34</td>
<td>3689.49</td>
</tr>
<tr>
<td>Age</td>
<td>4.32</td>
<td>2.19</td>
<td>0.73</td>
<td>11.39</td>
</tr>
<tr>
<td>Months bred</td>
<td>3.45</td>
<td>2.81</td>
<td>0.00</td>
<td>7.13</td>
</tr>
<tr>
<td>Percent of pens artificially inseminateda</td>
<td>10%</td>
<td>0.29</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Real cull cow price ($/cwt)</td>
<td>59.53</td>
<td>18.41</td>
<td>40.94</td>
<td>105.25</td>
</tr>
<tr>
<td>Lot size (head)</td>
<td>2.47</td>
<td>0.56</td>
<td>1.00</td>
<td>4.00</td>
</tr>
<tr>
<td>Number of lots</td>
<td>24</td>
<td>4.48</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>Percentage of lots mixedb</td>
<td>4%</td>
<td>0.200</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Percentage of lots Angusc</td>
<td>79%</td>
<td>0.406</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Percentage of lots Gelbviehd</td>
<td>14%</td>
<td>0.344</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Percentage of lots Balancer e</td>
<td>3%</td>
<td>0.168</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

**Note(s):** a 1 if female was AI-sired  
  b 1 if lot was defined as mixed  
  c 1 if lot was defined as Angus  
  d 1 if lot was defined as Gelbvieh  
  e 1 if lot was defined as Balancer

Table 1. Summary statistics of female lots sold from 2009 to 2018 at the University of Tennessee Crossville Plateau Research and Education Center (number of lots was 241).
ranged from open (zero) to seven months pregnant. The average animal was between three and four months pregnant. Only 10% of the lots sold were pregnant using AI-sired. Most of the lots sold were black Angus (79%) with Gelvieh being the second most sold breed (14%). There were also some lots sold that were Balancer cattle (3%) and lots including multiple breeds (4%), which we refer to as mixed lots.

**Simulation**

*Boyer et al. (2020)* used data from a spring-calving herd located at the Ames Plantation Research and Education Center, near Grand Junction, Tennessee, spanning from 1990 to 2008 to estimate calf weaning weight as a function of dam age and calf sex. These data have also been used by *Henry et al. (2016)* to compare spring- and fall-calving herds. More information about these herds’ management can be found in those papers as well as in *Campbell et al. (2013)*.

Production costs on a per head basis came from the University of Tennessee Extension Livestock Budgets (*University of Tennessee, 2019*). Total annual variable costs for the spring-calving herds were $550 per head. If the female purchased was not pregnant, the cost of breeding and maintaining her was assumed to be $250/head (*University of Tennessee, 2019*). Monthly Tennessee price data for 500–600 lb steers and heifers were collected from 2000 to 2018 (*USDA AMS, 2018*). All steer and heifer prices were also adjusted into 2018-dollar values using the Implicit Gross Domestic Product Price Deflator (*US Department of Labor, 2018*). Most cow-calf producers using a defined calving season in the Southeastern United States follow a spring-calving season, beginning in January and ending around mid-March (*Campbell et al., 2013*). This means most of the calves are sold during the months of September, October and November. Therefore, we used an average of these monthly prices for the sale price of calves.

A discount rate ($R$) of 5.5% was used to calculate NPV. The terminal age was assumed to be 12 years old. Previous studies have used similar terminal ages ranging from 10 to 12 years old (*Matthews and Short, 2001; Trapp, 1986; Mackay et al., 2004; Ibendahl et al., 2004; Boyer et al., 2020*). For calving rates, we assumed the calving rate was 85% for heifers and two-year old cows; 90% for three- to nine-year old cows; 85% 10- and 11-years cows and 80% for a 12-year old cow. This is a similar calving rate trend used in other studies (*Núñez-Dominguez et al., 1991; Ibendahl et al., 2004*). A shortcoming of this study is not having quality data to incorporate cow and calf death loss across scenarios.

**Results**

Parameter estimates for the hedonic pricing models are shown in Table 2. Heteroscedasticity was detected in the data across years, breed and lot size. Therefore, results were estimated using multiplicative heteroscedasticity in the variance equation which corrects for unequal variances. The parameter estimates for age, months pregnant, cull cow price and breed were significant at the 5% level and the signs of the estimated parameters were like previous studies.

Sale prices increased at a decreasing rate for age and months pregnant, but the interaction of these variables was not significant. Parameter estimates indicate prices were the highest for females between four and five months pregnant and when females were between four and five years old. *Mitchell et al. (2018)* also showed that the sale price of bred cows in Oklahoma increased until they reached five years old and then started decreasing. However, the length into pregnancy that maximized price was lower for our study than what *Mitchell et al. (2018)* reported. They found cows prices increased until they were eight months pregnant.
Females that were AI-sired pregnancies were not found to significantly increase sale price. This is different from previous work on sale prices of bred heifers, which showed AI-sired heifers were more valuable than natural service pregnancies (Parcell et al., 2006).

However, increasing cull cow prices at the time of purchase was found to increase the price of these females, which is what Mitchell et al. (2018) reported. Finally, relative to lots that mixed multiple breeds, Angus lots brought $127/head more and Gelbvieh lots brought $135/head more. This shows sellers of replacement females could increase sale price by marketing five-year-old, four month pregnant and uniform lots of replacement females.

Figure 1 shows the expected predicted prices of females across various age and months pregnant. Sale prices seemed to be grouped by months pregnant with four to five months pregnant having the highest sale prices, followed by three and six months pregnant. Next, sale prices were similar for two to seven-month pregnant females. One month pregnant and non-pregnant females were sold for the lowest prices with open females bringing the lowest sale price. These price differences across age and months pregnant could demonstrate real-world risks and returns a producer must consider in making this decision. Purchasing an open female is cheaper, but the lower price reflects the producers’ cost of developing and breeding as well as risk of pregnancy. It also displays the premiums producers place on

### Table 2.
Hedonic pricing model parameter estimates for purchase price ($/head) for female beef cattle lots sold from 2009 to 2018 at the University of Tennessee Crossville Plateau Research and Education Center (number of lots was 241)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Parameter estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>6.0718***</td>
</tr>
<tr>
<td>Months pregnant ($β_1$)</td>
<td>0.1663***</td>
</tr>
<tr>
<td>Months pregnant squared ($β_2$)</td>
<td>-0.0194***</td>
</tr>
<tr>
<td>Age ($β_3$)</td>
<td>0.0591***</td>
</tr>
<tr>
<td>Age squared ($β_4$)</td>
<td>-0.0065***</td>
</tr>
<tr>
<td>Artificial inseminated ($β_5$)</td>
<td>0.0001</td>
</tr>
<tr>
<td>Cull cow price ($β_6$)</td>
<td>-0.0074</td>
</tr>
<tr>
<td>Angus ($β_7$)*</td>
<td>0.01443***</td>
</tr>
<tr>
<td>Gelbvieh ($β_8$)*</td>
<td>0.08396***</td>
</tr>
<tr>
<td>Balancer ($β_9$)*</td>
<td>0.08889***</td>
</tr>
<tr>
<td></td>
<td>0.0359</td>
</tr>
</tbody>
</table>

**Note(s):** Asterisks (*** denote significance at the 0.01 level

* Reference for breed was mixed lot

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![Figure 1](image.png)

Predictive sale prices for female beef cattle sold ($/head) from 2009 to 2018 at the university of Tennessee Crossville Plateau Research and Education Center
certainty that comes with age. A five-year old cow could be sold for a higher price than a heifer because the older female has a track record of weaning a calf (Mitchell et al., 2018).

Simulation
Simulation results are shown in Table 3. Among several results, expected NPV was highest when an open heifer was purchased and sold as a pregnant nine-year-old ($559/head) and the lowest NPV occurred when a pregnant four-year old was purchased and sold a year later as open (-$567/head). For all scenarios, expected NPV was highest when a heifer was purchased and lowest when a four-year old cow was purchased. Between these extremes there are several profitable and unprofitable replacement decisions.

When an open female was purchased, expected NPV was positive when the female was sold as pregnant regardless of the age. Interestingly, selling a nine-year old pregnant female resulted in higher expected NPV than selling her as an open 12-year old. Selling a pregnant female at five, seven, or nine years old, when purchased open, was always more profitable than selling her as an open cow at age 12, regardless of purchased age. This shows that a higher salvage value for sale of a pregnant female before the end of her productive life increased NPV more than the additional calves produced if sold at the end of her productive life assumed in this model.

NPV was negative for some scenarios when a purchased open female was sold as open. This is likely explained by the females not producing enough calves to breakeven or be profitable. Boyer et al. (2020) showed the payback period on beef cows is between six and eight calves weaned, which supports why the NPV is negative for these females. This is an important conclusion that demonstrates the impact of reproductive failure on the long-term profitability of a herd. For example, if a two-year open cow is purchased and then sold as an open five-year old, it is due to reproductive failure. The producer may have sold three calves from her before she was sold, but overall, the return on this investment was negative.

When a pregnant female was purchased, a similar result was found that a producer could increase their NPV by selling this female as pregnant rather than as an open 12-year old. However, expected NPV was negative for several scenarios when a pregnant female was purchased. For example, a purchased pregnant heifer that was sold as open before 12 years old would result in a negative expected NPV. This further demonstrates the impact of reproductive failure on profitability. Purchased pregnant heifers and cows can have a positive NPV but are smaller than a purchased open heifer. Overall, buying an open female has a higher expected NPV than buying a pregnant female.

These results are helpful but standard deviations in Table 3 show risk will vary across these scenarios. Table 4 shows the probability of NPV being positive, or greater than zero, for all these scenarios. While selling a nine-year old pregnant cow was the most profitable scenario regardless of age, the likelihood of having a positive NPV was higher for other scenarios. For example, when a heifer was purchased either open or pregnant, NPV was most likely to be positive when this female was sold as a 7-year old pregnant cow. For cows, buying an open cow and selling her as a pregnant 5-year old would be the scenario most likely to have a positive NPV. These findings demonstrate the tradeoff between risk and profits for all these scenarios.

Risk analysis
First- and second-degree stochastic dominance showed no dominant scenario. SERF was used to determine the preferred scenario across risk aversion levels. There are too many scenarios to present in a figure, thus, we summarize the results by presenting the risk premiums for all scenarios that were preferred over the risk aversion levels (Figure 2). A risk-neutral (ARAC = 0) producer (or profit-maximizer) would purchase an open heifer and sell a nine-year old pregnant cow. However, as risk aversion increases the preferred scenario was to
<table>
<thead>
<tr>
<th>Age and pregnancy status when sold</th>
<th>Open</th>
<th>Pregnant</th>
<th>2-Year old</th>
<th>Open</th>
<th>Pregnant</th>
<th>3-Year old</th>
<th>Open</th>
<th>Pregnant</th>
<th>4-Year old</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>Open</td>
<td>-51 (415)</td>
<td>-299 (341)</td>
<td>-97 (309)</td>
<td>-367 (238)</td>
<td>-187 (197)</td>
<td>-437 (140)</td>
<td>-294 (86)</td>
<td>-567 (94)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>410 (503)</td>
<td>161 (419)</td>
<td>364 (397)</td>
<td>86 (310)</td>
<td>273 (285)</td>
<td>-12 (196)</td>
<td>180 (173)</td>
<td>-112 (85)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Open</td>
<td>94 (622)</td>
<td>-154 (547)</td>
<td>48 (516)</td>
<td>-221 (440)</td>
<td>-42 (404)</td>
<td>-327 (331)</td>
<td>-120 (292)</td>
<td>-413 (226)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>520 (702)</td>
<td>272 (620)</td>
<td>484 (596)</td>
<td>212 (511)</td>
<td>385 (483)</td>
<td>98 (397)</td>
<td>306 (370)</td>
<td>-18 (283)</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Open</td>
<td>186 (809)</td>
<td>-62 (733)</td>
<td>139 (703)</td>
<td>-129 (626)</td>
<td>49 (591)</td>
<td>-236 (514)</td>
<td>-14 (479)</td>
<td>-307 (404)</td>
</tr>
<tr>
<td>Pregnant</td>
<td>559 (878)</td>
<td>311 (798)</td>
<td>537 (712)</td>
<td>265 (689)</td>
<td>465 (659)</td>
<td>137 (575)</td>
<td>359 (546)</td>
<td>20 (461)</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Open</td>
<td>343 (1,069)</td>
<td>96 (991)</td>
<td>297 (936)</td>
<td>28 (883)</td>
<td>206 (851)</td>
<td>-78 (770)</td>
<td>100 (738)</td>
<td>-195 (657)</td>
</tr>
</tbody>
</table>

**Note(s):** Standard deviations are shown in parentheses.

Table 3. Expected net present values ($/head) for buying and selling beef cattle replacement females at various ages and pregnancy status.
Table 4. Probability of a positive expected net present value (\$/head) for buying and selling beef cattle replacement females at various ages and pregnancy status.

<table>
<thead>
<tr>
<th>Age and pregnancy status when sold</th>
<th>2-Year old</th>
<th>3-Year old</th>
<th>4-Year old</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pregnant</td>
<td>Pregnant</td>
<td>Pregnant</td>
</tr>
<tr>
<td>5 Open</td>
<td>45%</td>
<td>19%</td>
<td>38%</td>
</tr>
<tr>
<td>Pregnant</td>
<td>54%</td>
<td>65%</td>
<td>82%</td>
</tr>
<tr>
<td>7 Open</td>
<td>56%</td>
<td>39%</td>
<td>54%</td>
</tr>
<tr>
<td>Pregnant</td>
<td>79%</td>
<td>67%</td>
<td>79%</td>
</tr>
<tr>
<td>9 Open</td>
<td>59%</td>
<td>47%</td>
<td>58%</td>
</tr>
<tr>
<td>Pregnant</td>
<td>77%</td>
<td>65%</td>
<td>75%</td>
</tr>
<tr>
<td>12 Open</td>
<td>63%</td>
<td>54%</td>
<td>62%</td>
</tr>
</tbody>
</table>
purchase a two-year old open cow and sell a seven-year old pregnant cow. A slightly more risk averse producer would prefer to purchase this same type of female but sell her at age five. The optimal scenario changes to buying a three-year old open cow and selling her as a five-year old pregnant cow, then an extremely risk-averse producer would prefer to purchase a four-year old open cow and selling her as five-year pregnant cow.

When considering risk, optimal scenarios shift from the riskier purchase of heifers to more established cows, and as risk increases, a producer’s optimal decision would be to purchase an older female. Moreover, as risk increases, a producer’s preference is to sell older cows earlier. While holding on to them would produce a higher NPV, the risk associated with keeping these females increases. Relative to the profit maximizing producer, the extremely risk averse producer would need to be compensated between $400–470/head to switch from the less risky option of purchasing the 4-years cow to buying the open heifer. Lenders who evaluate risk of loans might also favor the less risky option of purchasing an older cow and keeping her for a shorter time period than buying a lower cost open heifer. That is, while the loan amount might be higher due to buying a more expense cow, this option is less risky than the smaller loan amount to purchase a heifer.

**Summary and conclusions**

The objective of this research was to determine the optimal age and pregnancy status for buying replacement beef females and determining when to sell them. We first estimate a hedonic pricing model to estimate how age, pregnancy status, breed and cull cow prices impact the sale price of these cattle. Data came from an annual heifer and cow sale in Crossville, Tennessee from 2009 to 2018. Then, a financial simulation model was developed to generate distributions of NPV for buying replacement females at various ages and pregnancy status and then selling that female at various ages and pregnancy status.

The hedonic pricing results show sale prices were highest for females between four and five months pregnant and prices increased until age five and then started decreasing. Results from the hedonic pricing model are consistent with previous studies and reveal productive insight into purchasing replacement females of various ages and pregnancy status. Premiums placed on these females by age and pregnancy status are consistent with different levels of risk associated with the scenario.

**Figure 2.** Utility weighted risk premiums from stochastic efficiency with respect to a function under a negative exponential utility function for buying and selling beef cattle replacement females at various ages and pregnancy status.
The simulation and risk analysis indicate buying an open female had a higher NPV than buying pregnant females regardless of the age when she was purchased. The model found purchasing a pregnant female was the most profitable when she was held to the end of her productive life. Selling an open female before the end of her productive life, who was purchased pregnant, will likely end up having a negative NPV. This demonstrates the negative economic impact of reproductive failure. Also, selling a female as pregnant before the end of her productive life increased her NPV. Purchasing a heifer resulted in a higher NPV than buying two through four-year-old cows. The optimal scenario changed across levels of risk aversion. As risk aversion increased, the optimal scenario was to purchase older open females and sell them pregnant before the end of their productive life.

Replacing a beef cow in a herd can be a complex but frequent decision for producers. This research offers unique insight on this decision by exploring how pregnancy status and age at selling impacts the investment while considering risk. These results have implications for educating producers as well as for lenders. A shortcoming of this study is that the sale data are from a November sale. This means the seasonal effects of price on replacements are not considered in this analysis. Future research could address this issue along with including the option of buying and selling cow-calf pairs and three-in-ones (pregnant cow with calf by side).

References
Appendix

The weaning weight response function was defined as

\[ y_{tk} = \beta_0 + \beta_1 A + \beta_2 A^2 + \beta_3 S + \nu_t + u_k + \epsilon_{tk} \]  

(4)

where \( y_{tk} \) is calf weaning weight (lb/head) for in year \( t \) from sire \( k \); \( A \) is age of the dam (year) when the calf was weaned; \( S \) is an indicator variable for sex (\( S = 1 \), steer; \( S = 0 \), heifer); \( \beta_0, \ldots, \beta_3 \) are coefficients to be estimated; \( \nu_t \sim N(0, \sigma^2_\nu) \) is the year random effect; \( u_k \sim N(0, \sigma^2_u) \) is the sire random effect; and \( \epsilon_{tk} \sim N(0, \sigma^2_\epsilon) \) is the random error term. Independence is assumed across all three random components. This equation was estimated using maximum likelihood with MIXED procedure in SAS 9.4. We tested weaning weights for heteroscedasticity with respect to cow age, year and sex using the likelihood ratio test. If heteroscedasticity was present, we report the results for the model that adjusts for the unequal variances.

Weaning weight response function parameters were drawn from the MVN distribution:

\[
\begin{bmatrix}
\hat{\beta}_0 \\
\vdots \\
\hat{\beta}_3
\end{bmatrix}
\sim
\text{MVN}
\left( 
\begin{bmatrix}
\hat{\beta}_0 \\
\vdots \\
\hat{\beta}_3
\end{bmatrix}, 
\begin{bmatrix}
\sigma^2_{\beta_0} & \cdots & \hat{\rho}_{\beta_0, \beta_1} \sigma_{\beta_0} \sigma_{\beta_1} \\
\vdots & \ddots & \vdots \\
\hat{\rho}_{\beta_3, \beta_0} \sigma_{\beta_3} \sigma_{\beta_0} & \cdots & \sigma^2_{\beta_3}
\end{bmatrix}
\right)
\]  

(5)
where “∼” denotes a randomly drawn parameter from the MVN distribution; the mean of the distribution is the vector of the estimated weaning weight response function coefficients \( \hat{\beta}_0, \ldots, \hat{\beta}_3 \); \( \sigma^2_{\hat{\beta}_0} \) are variance estimates of the parameters; and \( \rho_{\hat{\beta}_a} \sigma_{\hat{\beta}_a} \sigma_{\hat{\beta}_b} \) are estimated covariances between the parameters. Rho (\( \rho \)) is the correlation coefficient in the four-by-four covariance matrix of parameters.

Table A1 shows the coefficients for weaning weight response to age. Estimates for age were positive (\( p < 0.001 \)) and dam age squared were negative (\( p < 0.001 \)). This indicates weaning weights were increasing at a decreasing rate as age increased.

<table>
<thead>
<tr>
<th>Parameter estimates</th>
<th>Spring-calving season</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept (( \hat{\beta}_0 ))</td>
<td>353.48***</td>
</tr>
<tr>
<td>AGE (( \hat{\beta}_1 ))</td>
<td>65.529***</td>
</tr>
<tr>
<td>AGE(^2) (( \hat{\beta}_2 ))</td>
<td>-4.5218***</td>
</tr>
<tr>
<td>S (( \hat{\beta}_3 ))</td>
<td>32.723***</td>
</tr>
</tbody>
</table>

\( S^2 \) Log likelihood 16,150
AIC 16,164
BIC 16,171

\[ \text{Note(s)}: \text{Triple asterisks (***)} \text{ represent significance at the 1\% level} \]

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